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Application of Generalized EPR Entangled State in Quantum Teleportation

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Abstract We show how the generalized EPR entangled state $|\eta, \theta\rangle$ generated by an asymmetric beamsplitter can be directly applied to quantum teleportation to make detailed analysis and calculation. In the whole process of teleportation, the entanglement source is more flexible with the change of transmission and reflection coefficients of the beam splitter. When the squeezed state of $|\eta, \theta\rangle$ is used as quantum channel, the fidelity depends on the degrees of squeezing for entanglement and the reflection coefficient. Our calculation has been greatly simplified by using the Schmidt decomposition of $|\eta, \theta\rangle$.

Keywords Generalized EPR entangled state · Quantum teleportation · Asymmetric beamsplitter

1 Introduction

Quantum teleportation has drawn much attention in the field of quantum information technology because of its potential capability for transporting quantum or classical information. Quantum teleportation (QT), one of the most striking features in quantum information, can be realized based on entangled states. The first theoretical proposal of QT in the pioneer article of Bennett et al. [1] has stimulated experiments of QT using different types of Eintein-Podolsky-Rosen (EPR) pairs [2–5]. The idea was then generalized for continuous variable systems by using continuously entangled states [6], and a more practical scheme of continuous variable teleportation was proposed [7–9] in which a two-mode squeezed vacuum state is employed as an entangled state. Experimentally, teleportation of an optical field in a coherent state was demonstrated [2]. There is a considerable growth of interest in generalizing

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quantum information processing protocols to continuous variable systems [10, 11]. In this scheme of Ref. [11], the EPR entangled beams shared by sender and receivers are produced by mixing a pair of two-mode squeezed lights on one beamsplitter and separating them by using a polarizing beam splitter. Recently, for an asymmetric beam-splitter a new kind of entangled state $|\eta,\theta\rangle$ is introduced [12], which can make up a complete and orthonormal representation in two-mode Fock space.

The purpose of this paper is to show how the generalized EPR entangled state representation $|\eta,\theta\rangle$, whose generation can relies on the asymmetric beam splitter, can be directly used in discussing the quantum teleportation and what are the advantages of using such a state. We have derived the results of teleporting a single-mode state and a two-mode squeezed entangled state. When the squeezed state of $|\eta,\theta\rangle$ is used as quantum channel, the fidelity depends on the degrees of squeezing for entanglement and the reflection coefficient of the beam splitter. In the whole process of teleportation, the entanglement source is more flexible with the change of transmission and reflection coefficients which can be implemented experimentally. Our calculation has been greatly simplified by using the Schmidt decomposition of $|\eta,\theta\rangle$. Our work is arranged as follows: in Sect. 2 we briefly review the main properties of $|\eta,\theta\rangle$ and discuss the Schmidt decomposition of $|\eta,\theta\rangle$. In Sect. 3, the scheme of teleportation a single-mode state is proposed and the new squeezed state constructed by the representation $|\eta,\theta\rangle$ is also used as quantum channel to transfer single-mode state. Section 4 is devoted to discussing the teleportation of two-mode squeezed entangled vacuum state.

2 Bipartite Entangled State

In Ref. [12], for an asymmetric beam-splitter, the simultaneous eigenstate $|\eta,\theta\rangle$ (or $|\eta,\theta\rangle_{12}$) of commutative operators $(X_1\tan\theta - X_2)$ and $(P_1 + P_2\tan\theta)$ in two-mode Fock space (which we call the generalized EPR pair eigenvector with continuous variables) is constructed

$$\begin{aligned} |\eta,\theta\rangle = \exp\bigg[&-\frac{1}{2}|\eta|^2 + \eta a_1^\dagger - \eta^*(a_2^\dagger\sin 2\theta + a_1^\dagger\cos 2\theta) + \frac{1}{2}\eta^{*2}\cos 2\theta \\ &+ \frac{1}{2}(a_1^{\dagger 2} - a_2^{\dagger 2})\cos 2\theta + a_2^\dagger a_1^\dagger \sin 2\theta\bigg]|00\rangle, \end{aligned} \tag{1}$$

where $\eta = (\eta_1 + \mathtt{i}\eta_2)$ is a complex number, $|00\rangle$ is the two-mode vacuum state, $(a_i, a_i^\dagger)$, $i = 1, 2$, are two mode Bose annihilation and creation operators in Fock space, related to (X_i, P_i) by $X_i = (a_i + a_i^\dagger)/\sqrt{2}$, $P_i = (a_i - a_i^\dagger)/(\sqrt{2}\mathtt{i})$. The $|\eta,\theta\rangle$ states obey the eigenvector equations

$$(a_1 - a_2^\dagger\sin 2\theta - a_1^\dagger\cos 2\theta)|\eta,\theta\rangle = (\eta - \eta^*\cos 2\theta)|\eta,\theta\rangle, \tag{2}$$

$$(a_2 - a_1^\dagger\sin 2\theta + a_2^\dagger\cos 2\theta)|\eta,\theta\rangle = -\eta^*\sin 2\theta|\eta,\theta\rangle. \tag{3}$$

It then follows that

$$(X_1\tan\theta - X_2)|\eta,\theta\rangle = \sqrt{2}\eta_1\tan\theta|\eta,\theta\rangle, \tag{4}$$

$$(P_1 + P_2\tan\theta)|\eta,\theta\rangle = \sqrt{2}\eta_2\tan\theta|\eta,\theta\rangle, \tag{5}$$

where θ $(0 \le \theta \le \frac{\pi}{4})$ is related to the amplitude reflectivity and transmissivity of the asymmetric beam splitter. When $\theta = \pi/4$, which corresponds to a 50:50 beam-splitter, $|\eta, \pi/4\rangle$ reduces to $|\eta\rangle$ [13–15]. Therefore, $|\eta, \theta\rangle$ is a new entangled state with a non-trivial expression, we name it the generalized EPR eigenstate with continuous variables. The basic ingredient of the $|\eta, \theta\rangle$ state about the coordination-momentum entanglement can be demonstrated through its disentangling process, i.e.,

$$\left|\eta = \frac{\eta_1 + \mathrm{i}\eta_2}{\sqrt{2}}, \theta\right\rangle_{12} = \frac{e^{-\mathrm{i}\eta_1\eta_2/2}}{\sqrt{2}\cos\theta} \int_{-\infty}^{\infty} \mathrm{d}x |x\rangle_1 \otimes |(x - \eta_1)\tan\theta\rangle_2 e^{\mathrm{i}x\eta_2}, \tag{6}$$

$$\left|\eta = \frac{\eta_1 + \mathrm{i}\eta_2}{\sqrt{2}}, \theta\right\rangle_{12} = \frac{e^{-\mathrm{i}\eta_1\eta_2/2}}{\sqrt{2}\sin\theta} \int_{-\infty}^{\infty} \mathrm{d}p |p + \eta_2\rangle_1 \otimes |-p\cot\theta\rangle_2 e^{-\mathrm{i}p\eta_1}, \tag{7}$$

where $|x\rangle_i$ and $|p\rangle_i$ are the coordination eigenvector of X_i and the momentum eigenvector of P_i, respectively, i.e.,

$$|x\rangle_i = \pi^{-1/4} \exp\left(-\frac{x^2}{2} + \sqrt{2}x a_i^\dagger - \frac{1}{2} a_i^{\dagger 2}\right)|0\rangle_i, \tag{8}$$

$$|p\rangle_i = \pi^{-1/4} \exp\left(-\frac{p^2}{2} + \mathrm{i}\sqrt{2}p a_i^\dagger + \frac{1}{2} a_i^{\dagger 2}\right)|0\rangle_i. \tag{9}$$

Equations (6) and (7) are called Schmidt decomposition according to Ref. [16], which imply the entanglement and show that once particle 1 is measured in the state $|x\rangle_1$ (or $|p + \eta_2\rangle_1$), particle 2 is immediately collapses to the coordinate (or momentum) eigenstate $|(x - \eta_1)\tan\theta\rangle_2$ (or $|-p\cot\theta\rangle_2$), respectively. Furthermore, operating the operator $\exp(\mathrm{i}P_1 X_2 \cot\theta)$ on (6) state and using $\exp(\mathrm{i}P_i x_0)|x\rangle_i = |x - x_0\rangle_i$ and $X_i|x\rangle_i = x|x\rangle_i$, we have

$$\begin{aligned} e^{\mathrm{i}P_1 X_2 \cot\theta}|\eta, \theta\rangle_{12} &= \frac{e^{-\mathrm{i}\eta_1\eta_2/2}}{\sqrt{2}\cos\theta} \int_{-\infty}^{\infty} dx e^{\mathrm{i}P_1(x-\eta_1)}|x\rangle_1 \otimes |(x - \eta_1)\tan\theta\rangle_2 e^{\mathrm{i}x\eta_2} \\ &= \frac{e^{\mathrm{i}\eta_1\eta_2/2}}{\sqrt{2}\sin\theta}|\eta_1\rangle_1 \otimes \int_{-\infty}^{\infty} dx |x\rangle_2 e^{\mathrm{i}x\eta_2\cot\theta} \\ &= \frac{\sqrt{\pi}e^{\mathrm{i}\eta_1\eta_2/2}}{\sin\theta}|\eta_1\rangle_1 \otimes |p = \eta_2\cot\theta\rangle_2. \end{aligned} \tag{10}$$

That is to say,

$$|\eta, \theta\rangle_{12} = \frac{\sqrt{\pi}e^{\mathrm{i}\eta_1\eta_2/2}}{\sin\theta} e^{-\mathrm{i}P_1 X_2 \cot\theta}|\eta_1\rangle_1 \otimes |p = \eta_2\cot\theta\rangle_2, \tag{11}$$

which tells us that $e^{\mathrm{i}P_1 X_2 \cot\theta}$ is an entangling operator which entangles the coordinate eigenvector $|x = \eta_1\rangle_1$ and the momentum eigenvector $|p = \eta_2\cot\theta\rangle_2$ into the state $|\eta, \theta\rangle_{12}$. It implies that we can directly obtain the bipartite entangled state via the entangled operator $e^{\mathrm{i}P_1 X_2 \cot\theta}$ by virtue of linear optical implementation. From (6), (7) and (10) we see that the phase factor $e^{-\mathrm{i}\eta_1\eta_2/2}$ is essential for constructing the $|\eta, \theta\rangle_{12}$ state. Experimentally, furthermore, the $|\eta, \theta\rangle$ state can be generated by an asymmetric beam splitter as follows: letting the asymmetric beam splitter operator

$$S_2 = \exp[-2\theta(a_1^\dagger a_2 - a_2^\dagger a_1)] \tag{12}$$

operate on a pair of incoming modes $|p=0\rangle_1 \otimes |x=0\rangle_2$, where one is the zero-momentum eigenstate $|p=0\rangle_1 \sim \exp(\frac{1}{2}a_1^{\dagger 2})|0\rangle_1$ (maximum squeezing in the p-direction) and the other is the zero-position eigenstate $|x=0\rangle_2 \sim \exp(-\frac{1}{2}a_2^{\dagger 2})|0\rangle_2$ (maximum squeezing in x-direction), we have

$$\begin{aligned} &S_2|p=0\rangle_1 \otimes |x=0\rangle_2 \\ &= \exp\left[a_1^\dagger a_2^\dagger \sin 2\theta + \frac{1}{2}(a_1^{\dagger 2} - a_2^{\dagger 2})\cos 2\theta\right]|00\rangle \\ &= |\eta = 0, \theta\rangle. \end{aligned} \tag{13}$$

Then operating the displacement operator $D_1(\eta) \equiv \exp(\eta a_1^\dagger - \eta^* a_1)$ on (13) leads to (1), i.e.

$$D_1(\eta)\exp\left[a_1^\dagger a_2^\dagger \sin 2\theta + \frac{1}{2}(a_1^{\dagger 2} - a_2^{\dagger 2})\cos 2\theta\right]|00\rangle = |\eta, \theta\rangle, \tag{14}$$

where this displacement can be implemented by reflecting the light field of $|\eta = 0, \theta\rangle$ from a partially reflecting mirror (say 99% reflection and 1% transmission) and adding through the mirror a field that has been phase and amplitude modulated according to the value $\eta \equiv |\eta|e^{\mathrm{i}\varphi}$. Using the normal product form of two-mode vacuum state projector

$$|00\rangle\langle 00| =: \exp(-a_1^\dagger a_1 - a_2^\dagger a_2):, \tag{15}$$

the technique of integration within an ordered product (IWOP) of operators [17–19] we can smoothly prove the *completeness* relation

$$\begin{aligned} &\sin 2\theta \int \frac{\mathrm{d}^2\eta}{\pi} |\eta, \theta\rangle_{1212}\langle \eta, \theta| \\ &=: \exp\left\{\frac{1}{\sin^2 2\theta}\left[(a_1^\dagger - a_2 \sin 2\theta - a_1 \cos 2\theta)(a_1 - a_2^\dagger \sin 2\theta - a_1^\dagger \cos 2\theta)\right.\right. \\ &\quad \left. + \frac{1}{2}\cos 2\theta((a_1^\dagger - a_2 \sin 2\theta - a_1 \cos 2\theta)^2 + (a_1 - a_2^\dagger \sin 2\theta - a_1^\dagger \cos 2\theta)^2)\right] \\ &\quad \left. + (a_1^\dagger a_2^\dagger + a_1 a_2)\sin 2\theta + \frac{1}{2}(a_1^{\dagger 2} - a_2^{\dagger 2} + a_1^2 - a_2^2)\cos 2\theta - a_1^\dagger a_1 - a_2^\dagger a_2\right\}: \\ &=: e^0 := 1, \end{aligned} \tag{16}$$

where $\mathrm{d}^2\eta = \mathrm{d}\eta_1 \mathrm{d}\eta_2$, $::$ denotes normal ordering. The $|\eta, \theta\rangle$ state also possesses the *orthonormal* property

$$\langle \eta', \theta|\eta, \theta\rangle = \pi\delta(\eta_1 - \eta_1')\delta(\eta_2 - \eta_2')/\sin 2\theta, \ \ \eta = (\eta_1 + \mathrm{i}\eta_2). \tag{17}$$

According to Dirac's theory on representation in quantum mechanics, the set of $|\eta, \theta\rangle$ make up a new orthogonal and complete representation in the two-mode Fock space and can be viewed as Bell basis. Thus $|\eta, \theta\rangle$ can be either taken as an ideal quantum channel or an ideal quadrature phase measurement basis. Hence in this work we attempt to apply $|\eta, \theta\rangle$ to discuss quantum teleportation of continuous variable. We shall show that the $|\eta, \theta\rangle$ representation can help us to recapitulate the theory of quantum teleportation of continuous variables concisely. In the following we shall exhibit advantages of the usage of $|\eta, \theta\rangle$ state in four folds: (1) Teleported states can be calculated more explicitly, so one can come straight to the point. (2) Using the $|\eta,\theta\rangle$ representation the discussion of teleportation can be conveniently converted either to the coordinate-momentum representation or to the particle number representation. (3) Since the two-mode squeezing operator has a neat expression on the $|\eta,\theta\rangle$ basis, this approach is also concise when the quantum channel is a squeezed state. (4) This approach expounds teleportation theory in simple language by virtue of the $|\eta,\theta\rangle$ representation.

3 Teleportation of Single-Mode Quantum State

Let particle 1 and particle 2 be prepared in a generalized EPR entangled state $|\eta,\theta\rangle_{12}$, which shared by the sender "Alice" and the receiver "Bob", while Alice initially possess an unknown state $|\psi\rangle_3$. Thus the total initial state of this system is

$$|\psi\rangle_3 \otimes |\eta,\theta\rangle_{12}. \tag{18}$$

Alice wants to send this unknown state to Bob. For this purpose, she must perform a special measurement on particles 2 and 3 which projects them onto the generalized EPR entangled state $|\eta',\theta'\rangle_{23}$ ($|\eta',\theta'\rangle_{23}$ can be viewed as continuous Bell basis). According to (6) and (7), after the measurement the projected state for particle 1 is

$$\begin{aligned}
&{}_{23}\langle\eta',\theta'|\psi\rangle_3 \otimes |\eta,\theta\rangle_{12}\\
&= \frac{e^{\mathrm{i}\eta_1'\eta_2'/2}}{\sqrt{2}\cos\theta'}\int_{-\infty}^{\infty}\mathrm{d}x_2'\langle x'|\otimes_3\langle(x'-\eta_1')\tan\theta'|\psi\rangle_3 e^{-\mathrm{i}x'\eta_2'}\\
&\quad\otimes\frac{e^{-\mathrm{i}\eta_1\eta_2/2}}{\sqrt{2}\sin\theta}\int_{-\infty}^{\infty}\mathrm{d}p|p+\eta_2\rangle_1\otimes|-p\cot\theta\rangle_2 e^{-\mathrm{i}p\eta_1}\\
&= \frac{e^{\mathrm{i}\eta_1'\eta_2'/2-\mathrm{i}\eta_1\eta_2/2}}{2\sin\theta\cos\theta'}\int_{-\infty}^{\infty}\mathrm{d}x' e^{-\mathrm{i}x'\eta_2'}\frac{1}{\sqrt{2\pi}}\\
&\quad\times\int_{-\infty}^{\infty}\mathrm{d}p e^{-\mathrm{i}p(x'\cot\theta+\eta_1)}|p+\eta_2\rangle_{13}\langle(x'-\eta_1')\tan\theta'|\psi\rangle_3\\
&= \frac{e^{\mathrm{i}\eta_1'\eta_2'/2+\mathrm{i}\eta_1\eta_2/2}}{2\sin\theta\cos\theta'}\int_{-\infty}^{\infty}\mathrm{d}x' e^{\mathrm{i}x'(\eta_2\cot\theta-\eta_2')}{}_3\langle(x'-\eta_1')\tan\theta'|\psi\rangle_3|x'\cot\theta+\eta_1\rangle_1,
\end{aligned}\tag{19}$$

where we have used

$${}_i\langle x|p\rangle_i = \frac{1}{\sqrt{2\pi}}e^{\mathrm{i}px}, \qquad {}_i\langle p|X_i = -\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}p}{}_i\langle p|, \tag{20}$$

and the coefficient $2\sin\theta\sin\theta'$ is caused by the normalization condition in (17). For instance, if $|\psi\rangle_3 = |x\rangle_3$,

$$\begin{aligned}
{}_{23}\langle\eta',\theta'|x\rangle_3\otimes|\eta,\theta\rangle_{12} &= \frac{e^{\mathrm{i}(\eta_1'\eta_2'+\eta_1\eta_2)/2}}{2\sin\theta\cos\theta'}\int_{-\infty}^{\infty}\mathrm{d}x' e^{\mathrm{i}x'(\eta_2\cot\theta-\eta_2')}\delta(x-(x'-\eta_1')\tan\theta')\\
&\quad\times|x'\cot\theta+\eta_1\rangle_1
\end{aligned}$$

$$
\begin{aligned}
&= \frac{e^{\mathrm{i}(\eta_1'\eta_2'+\eta_1\eta_2)/2}}{2\sin\theta\sin\theta'} e^{-\mathrm{i}P_1\eta_1} e^{\mathrm{i}X_1(\eta_2\cot\theta-\eta_2')\tan\theta} e^{-\mathrm{i}P_1\eta_1'\cot\theta} \\
&\quad \times |x\cot\theta\cot\theta'\rangle_1 \\
&= \frac{e^{\mathrm{i}(\eta_1'\eta_2'+\eta_1\eta_2)/2}}{\sqrt{\sin 2\theta\sin 2\theta'}} U_1|x\rangle_1,
\end{aligned} \tag{21}
$$

where

$$U_1 = e^{-\mathrm{i}P_1\eta_1} e^{\mathrm{i}X_1(\eta_2\cot\theta-\eta_2')\tan\theta} e^{-\mathrm{i}P_1\eta_1'\cot\theta} S_1^{(1)}(\cot\theta\cot\theta'). \tag{22}$$

Noting (11) and (16), if we consider the generalized EPR entangled state as this one with a normalized factor $\sqrt{\sin 2\theta}$, then we can rewrite (21) as

$${}_{23}\langle\eta',\theta'|x\rangle_3 \otimes |\eta,\theta\rangle_{12} = e^{\mathrm{i}(\eta_1'\eta_2'+\eta_1\eta_2)/2} U_1|x\rangle_1, \tag{23}$$

and $S_1^{(1)}(\cot\theta\cot\theta')$ is a unitary operator and has its natural representation in coordinate basis, i.e.,

$$S_1^{(i)}(\mu) = \sqrt{\mu}\int_{-\infty}^{\infty} \mathrm{d}x\,|\mu x\rangle_{ii}\langle x|. \tag{24}$$

Hence up to a phase factor and a simple unitary transformation, Bob possesses the same state as the unknown state. If Alice now sends the results of the measurement (η',θ') to Bob via classical information channel, Bob can reproduce the unknown state by making a local operations to remove the phase and unitary transformation. Thus the teleportation is carried out successfully. For another example, if $|\psi\rangle_3 = |p\rangle_3$, using (11) and (19) we have

$${}_{23}\langle\eta',\theta'|p\rangle_3 \otimes |\eta,\theta\rangle_{12} \equiv e^{\mathrm{i}(\eta_1'\eta_2'+\eta_1\eta_2)/2} U_2|p\rangle_1, \tag{25}$$

where

$$U_2 \equiv e^{-\mathrm{i}P_1\eta_1} S_1(\cot\theta) e^{\mathrm{i}X_1(\eta_2\cot\theta-\eta_2't)} S_2^{(1)}(\tan\theta') e^{-\mathrm{i}P_1\eta_1'\tan\theta'},$$

and $S_2^{(1)}(\tan\theta')$ is a unitary operator and has its natural representation in momentum basis, i.e.,

$$S_2^{(i)}(\nu) = \sqrt{\nu}\int_{-\infty}^{\infty} \mathrm{d}p\,|\nu p\rangle_{ii}\langle p|. \tag{26}$$

After receiving the classical information (η',θ'), Bob makes a unitary transformation U^{-1} then up to phase factors in (25) the teleportation for the outcome state $|p\rangle_1$ is accomplished. Due to the completeness relation of $|x\rangle_3(|p\rangle_3)$, any single mode state $|\psi\rangle_3$ can be expressed as

$$|\psi\rangle_3 = \int_{-\infty}^{\infty} \mathrm{d}x\,|x\rangle_{33}\langle x|\psi\rangle_3 = \int_{-\infty}^{\infty} \mathrm{d}p\,|p\rangle_{33}\langle p|\psi\rangle_3, \tag{27}$$

thus any single mode state $|\psi\rangle_3$ can be teleported in this way. It is interesting to note that we can realize a perfect teleportation for an asymmetric beam splitter. On the other hand, we can modulate the unitary transformation performed by the receiver through switching the θ angle. Thus the entanglement source is flexible with the change of transmission and reflection coefficients which can be implemented experimentally.

Recently, the two-mode squeezed state have been applied to quantum teleportation of continuous variables [20, 21]. By noticing the fact that the new squeezing operator actually squeezes the generalized EPR state $|\eta,\theta\rangle$, we will show that the $|\eta,\theta\rangle$ representation also provides us with a feasible approach to derive the state teleported in their final form when the two-mode squeezed state is used as quantum channel shared by Alice and Bob. Let particle 1 and particle 2 share a squeezed state $|\tau\rangle_{12} = S_{12}(\tau)|00\rangle_{12}$, and Alice initially possesses an unknown state $|\psi\rangle_3$. Thus the initial state of the whole system is given by

$$|\psi\rangle_3 \otimes |\tau\rangle_{12} = |\psi\rangle_3 \otimes S_{12}(\tau)|00\rangle_{12}. \tag{28}$$

Alice wants to teleport $|\psi\rangle_3$ to Bob. For this purpose, she can make a joint Bell basis measurement on particles 2 and 3 which leads to another generalized EPR state, say $|\sigma\rangle = S_{23}(\sigma)|00\rangle_{23}$. After the measurement the projected state for particle 1 is

$$|\psi_{\text{out}}\rangle = {}_{23}\langle 00|S_{23}^{\dagger}(\sigma)|\psi_{\text{in}}\rangle_3 \otimes S_{12}(\tau)|00\rangle_{12}. \tag{29}$$

According to Ref. [12] the natural expression of the new two-mode squeezed operator is

$$\begin{aligned}
S_{12}(\tau) &= \sin 2\theta \int \frac{d^2\eta}{\tau\pi} |\eta/\tau,\theta\rangle_{1212}\langle\eta,\theta| \\
&= \frac{\sin 2\theta}{\sqrt{S}} \exp\left[\frac{1}{2S}\sinh^2\lambda\cos 2\theta(a_1^{\dagger 2} - a_2^{\dagger 2}) + \frac{1}{2S}a_1^{\dagger}a_2^{\dagger}\sinh 2\lambda\sin 2\theta\right] \\
&\quad \times\colon\exp\left[(a_1^{\dagger}, a_2^{\dagger})(M-1)\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}\right]\colon \\
&\quad \times \exp\left[\frac{1}{2S}\sinh^2\lambda\cos 2\theta(a_1^2 - a_2^2)\frac{1}{2S}a_1a_2\sinh 2\lambda\sin 2\theta\right],
\end{aligned} \tag{30}$$

where we have set $\tau = e^{\lambda}$ and $S = \cosh^2\lambda - \cos^2 2\theta$ and

$$M = \frac{\sin 2\theta}{S}\begin{pmatrix} \cosh\lambda\sin 2\theta & \sinh\lambda\cos 2\theta \\ -\sinh\lambda\cos 2\theta & \cosh\lambda\sin 2\theta \end{pmatrix}. \tag{31}$$

Equation (29) is worthy of paying attention since it indicates that the squeezing operator actually squeezes the generalized EPR eigenstate $|\eta,\theta\rangle$, which explains theoretically in a manifest fashion why the two-mode squeezed state itself belongs to the entangled state set. Equation (29) can provide us with a new approach for analyzing teleportation protocol with a squeezer. By using (30) and inserting the completeness relation of coherent state, after the measurement the projected state for particle 1 is

$$\begin{aligned}
|\psi_{\text{out}}\rangle &= \frac{\sin 2\theta\sin 2\theta'}{\sqrt{SS'}}{}_{23}\langle 00|\exp\left[\frac{1}{2S'}\sinh^2\mu\cos 2\theta'(a_2^2 - a_3^2)\right. \\
&\quad \left.+ \frac{a_2a_3}{2S'}\sinh 2\mu\sin 2\theta'\right]|\psi_{\text{in}}\rangle_3 \\
&\quad \times \int\frac{d^2z}{\pi}|z\rangle_{22}\langle z|\exp\left[\frac{1}{2S}\sinh^2\lambda\cos 2\theta(a_1^{\dagger 2} - a_2^{\dagger 2})\right. \\
&\quad \left.+ \frac{a_1^{\dagger}a_2^{\dagger}}{2S}\sinh 2\lambda\sin 2\theta\right]|00\rangle_{12} \\
&= \frac{\sin 2\theta\sin 2\theta'}{\sqrt{SS'}}{}_3\langle 0|\int\frac{d^2z}{\pi}\exp\left[-|z|^2 + \frac{za_3}{2S'}\sinh 2\mu\sin 2\theta' + \frac{z^*a_1^{\dagger}}{2S}\sinh 2\lambda\sin 2\theta\right]
\end{aligned}$$

$$\times \exp\left[\frac{\sinh^2\mu\cos 2\theta'}{2S'}(z^2 - a_3^2) + \frac{\sinh^2\lambda\cos 2\theta}{2S}(a_1^{\dagger 2} - z^{*2})\right]|\psi_{\mathrm{in}}\rangle_3 \otimes |0\rangle_1$$

$$= \frac{\sin 2\theta\sin 2\theta'}{\sqrt{\mathcal{D}}}{}_3\langle 0|\exp\left[\frac{1}{8\mathcal{D}SS'}(\mathcal{A}a_1^{\dagger 2} + \mathcal{B}a_1^\dagger a_3 - \mathcal{C}a_3^2)\right]|\psi_{\mathrm{in}}\rangle_3 \otimes |0\rangle_1 \tag{32}$$

where we have set $\sigma = e^\mu$, $S' = \cosh^2\mu - \cos^2 2\theta'$ and

$$\mathcal{A} = S'\sinh^2 2\lambda\sin^2 2\theta\sinh^2\mu\cos 2\theta', \tag{33}$$

$$\mathcal{B} = 2SS'\sinh 2\lambda\sin 2\theta\sinh 2\mu\sin 2\theta', \tag{34}$$

$$\mathcal{C} = S\sinh^2 2\mu\sin^2 2\theta'\sinh^2\lambda\cos 2\theta, \tag{35}$$

$$\mathcal{D} = SS' + \sinh^2\mu\cos 2\theta'\sinh^2\lambda\cos 2\theta, \tag{36}$$

and used the formula

$$\int \frac{d^2 z}{\pi}\exp[\zeta|z|^2 + \xi z + \eta z^* + fz^2 + gz^{*2}]$$

$$= \frac{1}{\sqrt{\zeta^2 - 4fg}}\exp\left[\frac{-\zeta\xi\eta + \xi^2 g + \eta^2 f}{\zeta^2 - 4fg}\right], \tag{37}$$

whose convergent condition is either

$$\mathrm{Re}(\zeta \pm f \pm g) < 0, \qquad \mathrm{Re}\left(\frac{\zeta^2 - 4fg}{\zeta \pm f \pm g}\right) < 0. \tag{38}$$

Based on (32) the teleportation can be analyzed more clearly. Letting $|\psi_{\mathrm{in}}\rangle_3$ be, for example, a vacuum state $|\psi\rangle_3 = |0\rangle_3$, we see that Bob gets the following output state, i.e.,

$$|\psi_{\mathrm{out}}\rangle = \frac{\sin 2\theta\sin 2\theta'}{\sqrt{\mathcal{D}}}\exp\left[\frac{\mathcal{A}}{8\mathcal{D}SS'}a_1^{\dagger 2}\right]|0\rangle_1, \tag{39}$$

which is a squeezed vacuum state in mode 1. Teleportation is usually quantified by the fidelity of the process. Fidelity is a measurement of the overlap of the input and output states. For a pure quantum state $|\psi_{\mathrm{in}}\rangle_3$, the fidelity is defined by

$$F = |\langle\psi_{\mathrm{in}}|\psi_{\mathrm{out}}\rangle|^2 = \frac{\sin^2 2\theta\sin^2 2\theta'}{\mathcal{D}}. \tag{40}$$

The fidelity of (40) is plotted in Fig. 1 as a function of the parameter λ and the angle θ. For another example, $|\psi_{\mathrm{in}}\rangle_3 = \exp(-\frac{|\alpha|^2}{2} + \alpha a_3^\dagger)|0\rangle_3$, i.e., a coherent state, from (32) we have

$$|\psi_{\mathrm{out}}\rangle = \frac{\sin 2\theta\sin 2\theta'}{\sqrt{\mathcal{D}}}\exp\left[\frac{1}{8\mathcal{D}SS'}(\mathcal{A}a_1^{\dagger 2} + \mathcal{B}a_1^\dagger\alpha - \mathcal{C}\alpha^2) - \frac{|\alpha|^2}{2}\right]|0\rangle_1, \tag{41}$$

and the fidelity is

$$F = \frac{\sin^2 2\theta\sin^2 2\theta'}{\mathcal{D}}\exp\left[\frac{1}{4\mathcal{D}SS'}(\mathcal{A}\,\mathrm{Re}\,\alpha^{*2} + \mathcal{B}|\alpha|^2 - \mathcal{C}\,\mathrm{Re}\,\alpha^2) - 2|\alpha|^2\right]. \tag{42}$$

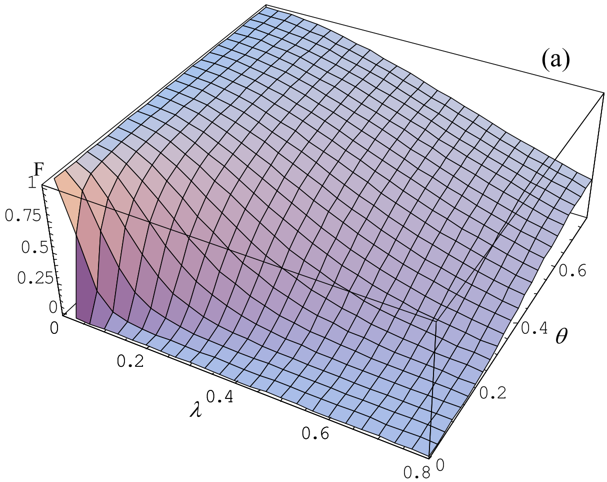

Fig. 1 The fidelity of teleportation F for the input vacuum state is plotted. We have set the squeezing parameters $\mu=\lambda$ and the angles $\theta=\theta'$. λ and θ are varied from 0 to 0.8 and from 0 to $\frac{\pi}{4}$, respectively

From (42) we see that the fidelity depends on the degrees of squeezing for entanglement and the reflection coefficient of the beam splitter (see Figs. 2 and 3). Especially, when $\theta=\theta'=\frac{\pi}{4}$, the output state in (41) is

$$|\psi_{\text{out}}\rangle=\frac{1}{\cosh\mu\cosh\lambda}\exp\left(-\frac{|\alpha|^2}{2}+a_1^\dagger\alpha\tanh\mu\tanh\lambda\right)|0\rangle_1, \tag{43}$$

which implies that (43) goes to $|\alpha\rangle_1$ in the limit of high squeezing, $\sigma\to\infty$, $\tau\to\infty$. Thus the initial state is teleported to particle 1. The fidelity in (42) reduces to

$$F_{\theta=\frac{\pi}{4}}=\frac{1}{\cosh^2\mu\cosh^2\lambda}\exp[-2|\alpha|^2(1-\tanh\mu\tanh\lambda)]. \tag{44}$$

This indicates that if the generalized EPR squeezed state is used as quantum channel state, the input state, say both vacuum state $|0\rangle$ and coherent state $|\alpha\rangle$, can not be teleported faithfully, i.e., with unit fidelity and unit probability and the fidelity depends on the degrees of squeezing for entanglement and the reflection coefficient of the beam splitter.

4 Teleportation of Two-Mode Quantum State

In this section, we consider how to teleport two-mode entangled state using $|\eta,\theta\rangle$ as quantum channels. Let particles 1 and 3 be prepared in a generalized state $|\eta,\theta\rangle_{13}$ and particles 2 and 4 prepared in a generalized state $|\eta',\theta'\rangle_{24}$. Alice and Bob share the two entangled states $|\eta,\theta\rangle_{13}$ and $|\eta',\theta'\rangle_{24}$ (two quantum channels). Initially, the unknown two-mode entangled state $|\psi\rangle_{56}$ is in mode 5 and 6, which will be teleported from Alice to Bob. Thus the initial state of the complete system is given by the directly product of the six-mode

$$|\psi\rangle_{56}\otimes|\eta,\theta\rangle_{13}\otimes|\eta',\theta'\rangle_{24}. \tag{45}$$

The scheme of teleportation can be described as follows. Firstly, Alice must make joint continuous Bell measurements (quadrature phase measurement) of X_3-X_5, and P_3+P_5, i.e., $|\eta''\rangle_{35}$ on particles 3 and 5; and X_4-X_6, and P_4+P_6, i.e., $|\eta'''\rangle_{46}$ on particles 4 and 6.

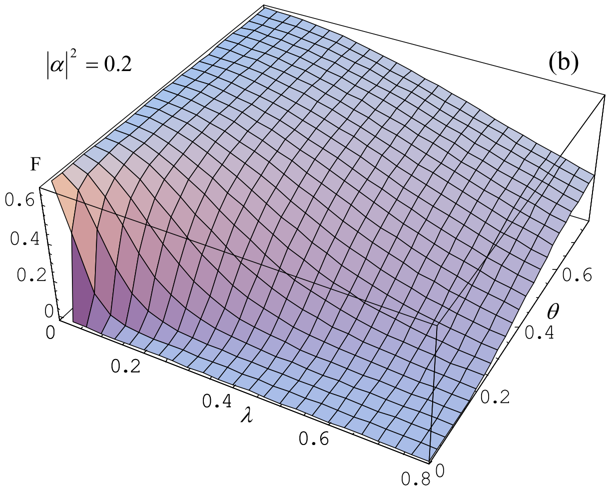

Fig. 2 The fidelity of teleportation F for the input coherent state is plotted. The displacing parameter $|\alpha|^2$ is fixed to 0.2. We have set the squeezing parameters $\mu = \lambda$ and the angles $\theta = \theta'$. λ and θ are varied from 0 to 0.8 and from 0 to $\frac{\pi}{4}$, respectively

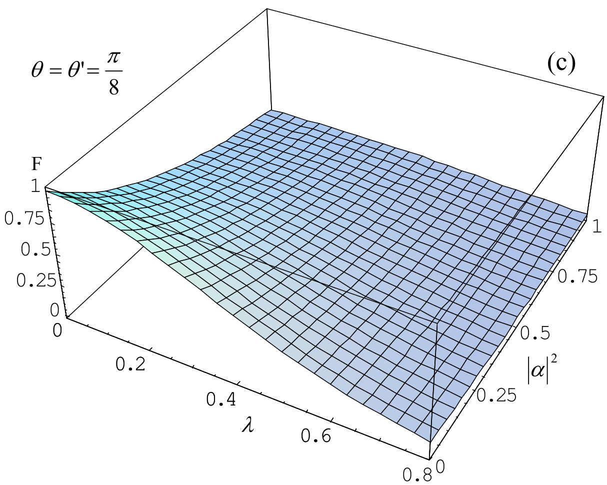

Fig. 3 The fidelity of teleportation F for the input coherent state is plotted. The angle parameter $\theta = \theta'$ is fixed to $\frac{\pi}{8}$. We have set the squeezing parameters $\mu = \lambda$. λ and $|\alpha|^2$ are varied from 0 to 0.8 and from 0 to 1, respectively

So the total projection state is $|\eta''\rangle_{35} \otimes |\eta'''\rangle_{46}$. After the measurement the projected state for particles 1 and 2 is

$$\begin{aligned}
|\Psi\rangle &\equiv {}_{46}\langle\eta'''| \otimes_{35} \langle\eta''|\psi\rangle_{56} \otimes |\eta,\theta\rangle_{13} \otimes |\eta',\theta'\rangle_{24} \\
&= \frac{A}{2\cos\theta\cos\theta'} \int_{-\infty}^{\infty} \mathrm{d}x_4''' \langle x'''| \otimes {}_6\langle x''' - \eta_1'''| e^{-\mathrm{i}\eta_2''' x'''} \\
&\quad \otimes \int_{-\infty}^{\infty} \mathrm{d}x''{}_3 \langle x''| \cdot_5 \langle x'' - \eta_1''|\psi\rangle_{56} e^{-\mathrm{i}\eta_2'' x''} \\
&\quad \otimes \int_{-\infty}^{\infty} \mathrm{d}x |x\rangle_1 \otimes |(x-\eta_1)\tan\theta\rangle_3 e^{\mathrm{i}x\eta_2} \\
&\quad \otimes \int_{-\infty}^{\infty} \mathrm{d}x' |x'\rangle_2 \otimes |(x'-\eta_1')\tan\theta'\rangle_4 e^{\mathrm{i}x'\eta_2'}
\end{aligned}$$

$$= \frac{A}{2\cos\theta\cos\theta'}\int_{-\infty}^{\infty} \mathrm{d}x e^{\mathrm{i}x\eta_2} e^{-\mathrm{i}\eta_2''(x-\eta_1)\tan\theta} \int_{-\infty}^{\infty} \mathrm{d}x' e^{\mathrm{i}x'\eta_2'} e^{-\mathrm{i}\eta_2'''(x'-\eta_1')\tan\theta'}$$
$$\cdot_6 \langle (x'-\eta_1')\tan\theta' - \eta_1'''| \otimes {}_5\langle (x-\eta_1)\tan\theta - \eta_1''|\psi\rangle_{56}|x\rangle_1 \otimes |x'\rangle_2, \tag{46}$$

where

$$A = e^{\mathrm{i}(\eta_1'''\eta_2'''+\eta_1''\eta_2''-\eta_1\eta_2-\eta_1'\eta_2')/2}. \tag{47}$$

Based on (46) the teleportation can be analyzed more clearly. Now we are concerned with the situation when $|\psi\rangle_{56}$ is a two-mode squeezed entangled vacuum state

$$|\psi\rangle_{56} = \operatorname{sech}\gamma \exp(a_5^\dagger a_6^\dagger \tanh\gamma)|00\rangle_{56}, \tau = e^\gamma, \tag{48}$$

which has its natural expression in the $|\eta\rangle_{56}$ representation [22]

$$|\psi\rangle_{56} = \frac{1}{\tau}\int \frac{\mathrm{d}^2\eta}{\pi}|\eta/\tau\rangle_{56\,56}\langle\eta|00\rangle_{56} = \frac{1}{\tau}\int\frac{\mathrm{d}^2\zeta}{\pi}e^{-|\zeta|^2/2}|\zeta/\tau\rangle_{56}. \tag{49}$$

Using the Schmidt decomposition of $|\zeta/\tau\rangle_{56}$ in (6) (with $\theta = \frac{\pi}{4}$) we have

$$\begin{aligned}
&{}_6\langle (x'-\eta_1')\tan\theta' - \eta_1'''| \otimes_5 \langle (x-\eta_1)\tan\theta - \eta_1''|\zeta/\tau\rangle_{56} \\
&\quad = e^{-\mathrm{i}\zeta_1\zeta_2/2\tau^2}{}_6\langle (x'-\eta_1')\tan\theta' - \eta_1'''| \otimes_5 \langle (x-\eta_1)\tan\theta - \eta_1''| \\
&\qquad \times \int_{-\infty}^{\infty}\mathrm{d}x''|x''\rangle_5 \otimes |x''-\zeta_1/\tau\rangle_6 e^{\mathrm{i}x''\zeta_2/\tau} \\
&\quad = e^{-\mathrm{i}\zeta_1\zeta_2/2\tau^2}\delta[(x'-\eta_1')\tan\theta' - \eta_1''' - (x-\eta_1)\tan\theta + \eta_1'' + \zeta_1/\tau]e^{\mathrm{i}[(x-\eta_1)\tan\theta-\eta_1'']\zeta_2/\tau}.
\end{aligned} \tag{50}$$

Substituting (50) into (49) and then into (46), we can rewrite (46) as

$$\begin{aligned}
|\Psi\rangle &= \frac{A}{2\cos\theta\cos\theta'}\frac{1}{\tau}\int\frac{\mathrm{d}^2\zeta}{\pi}e^{-|\zeta|^2/2}e^{-\mathrm{i}\zeta_1\zeta_2/2\tau^2} \\
&\quad \times \int_{-\infty}^{\infty}\mathrm{d}x e^{\mathrm{i}x\eta_2}e^{-\mathrm{i}\eta_2''(x-\eta_1)\tan\theta}e^{\mathrm{i}[(x-\eta_1)\tan\theta-\eta_1'']\zeta_2/\tau} \\
&\quad \times \int_{-\infty}^{\infty}\mathrm{d}x' e^{\mathrm{i}x'\eta_2'}e^{-\mathrm{i}\eta_2'''(x'-\eta_1')\tan\theta'} \\
&\quad \times \delta[x'\tan\theta' - (x-\eta_1)\tan\theta - \eta_1'\tan\theta' - \eta_1''' + \eta_1'' + \zeta_1/\tau]|x\rangle_1 \otimes |x'\rangle_2 \\
&= \frac{A\cot\theta'}{2\cos\theta\cos\theta'}e^{\mathrm{i}X_2\eta_2'}e^{-\mathrm{i}\eta_2'''(X_2-\eta_1')\tan\theta'}e^{\mathrm{i}X_1\eta_2}e^{-\mathrm{i}\eta_2''(X_1-\eta_1)\tan\theta} \\
&\quad \times \frac{1}{\tau}\int\frac{\mathrm{d}^2\zeta}{\pi}e^{-|\zeta|^2/2}e^{-\mathrm{i}\zeta_1\zeta_2/2\tau^2}\int_{-\infty}^{\infty}\mathrm{d}x e^{\mathrm{i}[(x-\eta_1)\tan\theta-\eta_1'']\zeta_2/\tau} \\
&\quad \times |x\rangle_1 \otimes |[(x-\eta_1)\tan\theta + \eta_1'\tan\theta' + \eta_1''' - \eta_1'' - \zeta_1/\tau]\cot\theta'\rangle_2 \\
&= \frac{A}{\sqrt{\sin 2\theta\sin 2\theta'}}e^{\mathrm{i}X_2\eta_2'}e^{-\mathrm{i}\eta_2'''(X_2-\eta_1')\tan\theta'}e^{\mathrm{i}X_1\eta_2}e^{-\mathrm{i}\eta_2''(X_1-\eta_1)\tan\theta} \\
&\quad \times e^{-\mathrm{i}P_2(\eta_1'\tan\theta'+\eta_1''')\cot\theta'}S_1^{(2)}(\cot\theta')e^{-\mathrm{i}\eta_1 P_1}S_1^{(1)}(\cot\theta)e^{-iP_1\eta_1''}
\end{aligned}$$

$$
\begin{aligned}
&\times \frac{1}{\tau}\int \frac{\mathrm{d}^2\zeta}{\pi} e^{-|\zeta|^2/2} e^{-\mathrm{i}\zeta_1\zeta_2/2\tau^2} \int_{-\infty}^{\infty} \mathrm{d}y e^{\mathrm{i}y\zeta_2/\tau} |y\rangle_1 \otimes |y-\zeta_1/\tau\rangle_2 \\
&= \frac{A}{\sqrt{\sin 2\theta \sin 2\theta'}} e^{\mathrm{i}(\eta_1'\eta_2''' \tan\theta' + \eta_1\eta_2'' \tan\theta)} e^{\mathrm{i}[X_2(\eta_2' - \eta_2''' \tan\theta') + X_1(\eta_2 - \eta_2'' \tan\theta)]} \\
&\quad \times e^{-\mathrm{i}P_2(\eta_1' \tan\theta' + \eta_1''')\cot\theta'} S_1^{(2)}(\cot\theta') e^{-\mathrm{i}\eta_1 P_1} S_1^{(1)}(\cot\theta) e^{-iP_1\eta_1''} |\psi\rangle_{12}.
\end{aligned} \tag{51}
$$

Noting (11) and (16) we can further rewrite (51) as

$$
\begin{aligned}
|\Psi\rangle = {}& A e^{\mathrm{i}(\eta_1'\eta_2''' \tan\theta' + \eta_1\eta_2'' \tan\theta)} e^{\mathrm{i}[X_2(\eta_2' - \eta_2''' \tan\theta') + X_1(\eta_2 - \eta_2'' \tan\theta)]} \\
& \times e^{-\mathrm{i}P_2(\eta_1' \tan\theta' + \eta_1''')\cot\theta'} S_1^{(2)}(\cot\theta') e^{-\mathrm{i}\eta_1 P_1} S_1^{(1)}(\cot\theta) e^{-\mathrm{i}P_1\eta_1''} |\psi\rangle_{12}.
\end{aligned} \tag{52}
$$

Comparing (52) with (48), we see that up to the inessential factor and a unitary transform, the outcome state in modes 1 and 2 is the same as the incoming state $|\psi\rangle_{56}$. After Alice informs Bob of the data of (η'', η''') through a classical channel, Bob then make the unitary transformation to obtain the unknown squeezed entangled state. That is to say, we can perform the perfect teleportation of two-mode squeezed state via the quantum channels $|\eta, \theta\rangle$ and local unitary operations. For continuous variable teportation, a description has been presented recently in terms of characteristic functions, which is particularly suited when dealing with non-Gaussian states and resources [23, 24].

In summary, we introduce the generalized EPR entangled state $|\eta, \theta\rangle$ as quantum channel to teleport quantum information. We show how $|\eta, \theta\rangle$ whose generation can relies on the asymmetric beam splitter can be directly applied to quantum teleportation to make detailed analysis and calculation. We have derived the results of teleporting a single-mode state and a two-mode squeezed entangled state using $|\eta, \theta\rangle$ or its squeezed state and via $|\eta, \theta\rangle$, respectively. When the squeezed state of $|\eta, \theta\rangle$ is used as quantum channel, the fidelity depends on the degrees of squeezing for entanglement and the reflection coefficient of the beam splitter. In the whole process of teleportation, the entanglement source is more flexible with the change of transmission and reflection coefficients which can be implemented experimentally. Our calculation has been greatly simplified by using the Schmidt decomposition of $|\eta, \theta\rangle$.

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